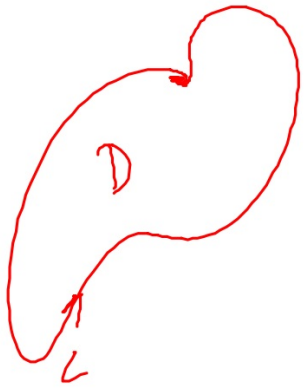
G.T:

$$\oint_C f\,dx + g\,dy = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA.$$



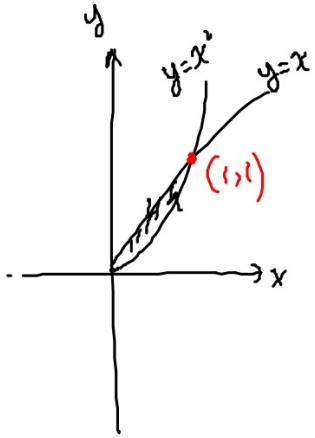
$$\text{Circulation} = \oint_C M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$\text{Flux} = \oint_C M\,dy - N\,dx = \iint_D \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA.$$

15)

$\vec{F} = \underbrace{xy}_{M}\vec{i} + \underbrace{y^2}_{N}\vec{j}$.

$$\text{Circulation} = \oint_C M\,dx + N\,dy \overset{G.T}{=} \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$



$$= \int_0^1 \int_{x^2}^{x} -x\,dy\,dx = \int_0^1 (x^3 - x^2)\,dx = \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}.$$

$$\text{Flux} = \oint_C M\,dy - N\,dx \overset{G.T}{=} \iint_D \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA$$

$$= \int_0^1 \int_{x^2}^{x} 3y\,dy\,dx = \frac{3}{2}\int_0^1 (x^2 - x^4)\,dx = \frac{3}{2}\left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1$$

$$= \frac{3}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{3}{2}\cdot\frac{2}{15} = \frac{1}{5}.$$

#30

Given that area of D $= 100\ m^2$

Evaluate $I = \oint_C xy^2\,dx + (x^2y + 2x)\,dy$

G.T $\Rightarrow I = \iint_D (2xy + 2 - 2xy)\,dA$

$= \iint_D 2\,dA = 2\iint_D dA$

$= 200$